Ann Oper Res (2012) 192:49–66
DOI 10.1007/s10479-010-0768-0

Fighting strategies in a market with counterfeits

Jie Zhang · L. Jeff Hong · Rachel Q. Zhang

Published online: 11 July 2010

Abstract Counterfeiting is a widely spread phenomenon and has seen rapid growth in recent years. In this paper, we adopt the standard vertical differentiation model and allow consumers the choices of purchasing an authentic product, purchasing a counterfeit, or not buying. We focus on how non-deceptive counterfeits, which consumers know at time of purchase that the products are counterfeits with certainty, affect the price, market share and profitability of brand name products. We also consider the strategies for brand name companies to fight counterfeiting. We compare different fighting strategies in a market with one brand name product and its counterfeit, and derive equilibrium fighting strategies in a market with two competing brand name products and a counterfeit under general conditions.

1 Introduction

The global counterfeit business has been targeting everything from computer chips to life-saving medicines. The World Customs Organization estimates 7% of world merchandise, or \$512 billion in 2004, may be counterfeit products. The multinationals are spending tens of millions of dollars trying to stop counterfeiters. They hire full-time employees, investigators, lawyers, and informants for that purpose, invest on new technologies to authenticate their products, redesign packaging to make counterfeiting more difficult, keep altering the

Electronic supplementary material The online version of this article (doi:10.1007/s10479-010-0768-0) contains supplementary material, which is available to authorized users.

J. Zhang (✉) · L.J. Hong · R.Q. Zhang
Department of Industrial Engineering and Logistics Management, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, P.R. China
e-mail: jiezh@ust.hk

L.J. Hong
e-mail: hongl@ust.hk

R.Q. Zhang
e-mail: rzhang@ust.hk

look of products, dig through dumpsters at suspect factories looking for counterfeit packaging, and even raid factories (Balfour 2005). It is widely perceived that governments are not doing enough to crack down counterfeiting because most of the harm is inflicted on foreign brand owners, while the governments argue that they are doing what they can by prosecuting counterfeiters and raising the awareness of intellectual property issues. The fact is counterfeiting continues to spread. While the world trade is growing at 3–4% percent, the growth rate of counterfeits is around 150% (Smith 1997). This paper investigates the impact of counterfeits on the price, market share and profitability of authentic products and strategies for brand name companies to fight counterfeiting.

In international markets, four broad categories of products are more vulnerable to piracy (Jacobs et al. 2001): (1) highly visible, high volume, low-tech products with well-known brand names (e.g., candies and soft drinks); (2) high-priced, high-tech products (e.g., computer software, DVDs and auto parts); (3) exclusive, prestige products (e.g., well-known apparels, handbags and accessories); and (4) intensive R&D, high-tech products (e.g., pharmaceuticals).

All counterfeits can be divided into two categories, non-deceptive and deceptive, first defined by Grossman and Shapiro (1988a, 1988b). Non-deceptive counterfeits are those which consumers know at time of purchase that the products are counterfeits with little uncertainty due to the price, quality and location of sale. For instance, DVDs sold at a little over \$1, \$5 ROLEX watches, and \$20 Louis Vuitton handbags in some Chinese markets. These products usually have low performance risks as opposed to counterfeit medicines or auto parts, referred to as deceptive counterfeits that consumers believe to be authentic at time of purchase. They are usually sold at similar prices as and packaged to resemble the authentic ones, and can cause serious harm to consumers. In this paper, we focus on non-deceptive counterfeits.

Research on counterfeiting is fairly recent and mainly in the fields of marketing, information technology, and business ethics. Most studies are conceptual, providing frameworks for fighting counterfeiting without quantitative analysis; or empirical, sampling using questionnaires followed by data analysis to find statistical evidence that supports certain propositions.

The majority of the early work and activities taken by business firms are along the supply dimension. These include warning consumers of potential harm of counterfeits and raising public awareness of the importance of intellectual property protection (Harvey and Ronkainen 1985; Harvey 1987), establishing laws to make counterfeiting a criminal offense and raiding facilities that manufacture and sell counterfeit products (Bamossy and Scammon 1985; Bush et al. 1989; Carty 1994; Olsen and Granzin 1992; Onkvisit and Shaw 1989), and labeling original products to make it less vulnerable to piracy (Chaudhry and Walsh 1996). Jacobs et al. (2001) provide a summary of these protective measures and a structure leading to a whole prevention strategy. Green and Smith (2002) focus on brand counterfeiting and illustrate how International Spirits Distributors, a wholly-owned subsidiary of a major European alcoholic beverage producer, addresses the threat of counterfeiters in the lucrative Thailand market.

Bloch et al. (1993), Cordell et al. (1996) and Tom et al. (1998) are among the first in marketing to study the consumer side of the problem. By conducting field studies in malls and flea markets in the U.S., they find that over one-third of their samples indicated their willingness to purchase counterfeits. These studies suggest the existence of sizable demand for counterfeits in the U.S. Kwong et al. (2003) study the relationship between attitudes towards piracy and intention to buy pirated CDs using samples from some Chinese markets. Their findings reveal that a consumer's intention to buy pirated CDs is strongly affected by his or her attitude towards piracy. Social benefit of dissemination and anti-big business attitude have a positive relationship with intention to buy pirated CDs, while the social cost of piracy and ethical belief have a negative relationship. Eisend and Schuchert-Güler (2006) provide a review of a large body of empirical work that follows. They point out the lack of a general qualitative framework that can integrate the existing results consistently and build a cognitive-dissonance model to explain the counterfeit purchasing process.

There are only a few analytical studies on counterfeiting, mostly in economics and marketing. Grossman and Shapiro (1988a) investigate deceptive counterfeiting using vertical differentiation models. They show that a brand name company may either raise or lower the quality of its products to drive the counterfeits from foreign imports out of the market. However, changing the quality levels will also lead to the decrease of home country's welfare if there is no limit for home companies to enter the market. They also discuss government policies towards local non-deceptive counterfeiters (Grossman and Shapiro 1988b). Conner and Rumelt (1991) and Givon et al. (1995) show that software piracy actually has a strong promotional effect that may benefit an authentic product. Scandizzo (2001) considers quality improvement as patent race over time. Assuming that companies who do not enter the patent race become counterfeiters, he obtains the equilibrium number of counterfeiters and brings the distribution of consumers' income level into the model. He shows that the more skewed the distribution is towards the poor, the less the counterfeits affect the brand name company's profit but the greater benefit they bring to the social welfare. Assuming two possible quality levels for a brand name product, high or low, Qian (2006) shows that a brand name company will choose a higher quality level after counterfeits' entry if the additional cost is below a certain threshold as well as raising the price if the counterfeits' quality is low. She also studies the effects of both price and non-price signaling, and governments' roles in fighting deceptive counterfeiting under asymmetric information. An empirical study of Chinese shoe companies is provided. Jain (2008) investigates piracy from consumers' illegally copying and shows that brand name companies are better off without copyright protection even if it is costless under certain conditions.

In this paper, we focus on the impact of non-deceptive counterfeits on the price, market share and profitability of brand name products, and how brand name companies can go about fighting counterfeits. We adopt the vertical differentiation model (Mussa and Rosen 1978; Shaked and Sutton 1982) commonly used in economics and marketing, and allow consumers the choices of purchasing an authentic product, a counterfeit, or not buying. A consumer's utility towards a product is modeled as a function of the quality and price of the product. The quality of a product includes the actual quality as well as its brand value. We start with the market with a single brand name product and its counterfeit. As expected, a counterfeit lowers the price and profit of the brand name product, but more consumers will make purchases in the presence of a counterfeit. We also show that a brand name company has more incentive to invest in raising the quality level of its own product, referred to as the quality improvement strategy, than reducing that of a counterfeit, referred to as the direct fighting strategy. This explains why brand name companies are often reluctant to take on the counterfeits directly.

We then consider a market with two competing brand name products and a counterfeit. So far, few work has been done to study the effect of competition among the brand name companies in a market with counterfeits besides an argument in Conner and Rumelt (1991), due to difficulties in its analysis with three or more products. We show that the brand name product with a larger market share, referred to as the big brand, suffers a greater absolute loss but smaller relative loss. Thus, both the brand name companies are the victims of the counterfeit and have incentive to fight counterfeiting. With competition, intuitively, the brand name companies are even more reluctant to fight counterfeiting directly. However, increasing the quality of the small brand name product may intensify the competition between the two brand name products and it may be of the best interest of the small brand to fight counterfeiting directly. If reducing the quality level of the counterfeit is the only option, the small brand will simply rely on the big brand to make an investment and collect the benefit without making any investment unless the small brand is significantly more effective in reducing the quality of the counterfeit than the big brand. Under some conditions, there exists no or multiple equilibrium investment strategies, which call for cooperation between the brand name companies in fighting counterfeiting.

The paper is organized as follows. Section 2 presents the basic model with one brand name product and a non-deceptive counterfeit, and analyzes several strategies for the brand name company to protect itself from counterfeiting. Section 3 expands the model to a market with two competing products and a counterfeit. We summarize our conclusions in Sect. 4. The proofs of the propositions and derivations of some formulas can be found in the Appendix.

2 Basic model and analysis

We first consider a market served by a brand name product and a non-deceptive counterfeit. A consumer in the market has the option of purchasing the authentic product (choice a), purchasing a counterfeit (choice f), or not buying (choice 0). Throughout the paper, we will use choice and product interchangeably. A consumer's utility towards product i, $i = a, f$, is given by $u_i = \theta q_i - p_i$ and $u_0 = 0$ where $p_i > 0$ is the price of product i, a decision variable; $q_i > 0$ is the quality level of product i, and θ is uniformly distributed over [0, 1] representing consumer heterogeneity.

A consumer will make a purchase only if the utility of a product is nonnegative and will select a product with a higher utility. We assume that $q_a > q_f$, which is true in most applications and also assumed in Grossman and Shapiro (1988b) and Qian (2006). Then, $p_a > p_f$ in order for the counterfeit to have a nonnegative market share. Let $P_i(p_a, p_f)$ be the market share of product i, $i = a, f$, for a given (p_a, p_f). Then,

$$P_a(p_a, p_f) = \Pr(u_a \geq u_f,\ u_a \geq 0) = \Pr\left(\theta \geq \frac{p_a - p_f}{q_a - q_f}, \theta \geq \frac{p_a}{q_a}\right),$$

$$P_f(p_a, p_f) = \Pr(u_a < u_f,\ u_f \geq 0) = \Pr\left(\frac{p_f}{q_f} \leq \theta < \frac{p_a - p_f}{q_a - q_f}\right).$$

It is easy to verify that, for both products to exist in the market, $0 < \frac{p_f}{q_f} < \frac{p_a - p_f}{q_a - q_f} < 1$ and

$$P_a(p_a, p_f) = 1 - \frac{p_a - p_f}{q_a - q_f},$$

$$P_f(p_a, p_f) = \frac{p_a - p_f}{q_a - q_f} - \frac{p_f}{q_f}.$$

Otherwise, either $P_a(p_a, p_f) = 0$ if $\frac{p_a - p_f}{q_a - q_f} \geq 1$ or $P_f(p_a, p_f) = 0$ if $\frac{p_f}{q_f} \geq \frac{p_a - p_f}{q_a - q_f}$. Then, the portion of the consumers who will not make a purchase is given by $P_0(p_a, p_f) = \frac{p_f}{q_f}$.

As expected, the market share of either product decreases (increases) in its own price (quality level) and increases (decreases) in the price (quality level) of the other product.

If we normalize the size of the potential market to 1, for any given (p_a, p_f), the profit generated from product i, denoted as $\pi_i(p_a, p_f)$, $i = a, f$, can be written as

$$\pi_a(p_a, p_f) = (p_a - c_a)P_a(p_a, p_f),$$
$$\pi_f(p_a, p_f) = p_f P_f(p_a, p_f)$$

where c_a is the variable cost associated with the authentic product. We ignore the variable cost of the counterfeit product and possible fixed costs associated with each product as in Qian (2006) as the variable cost of a counterfeit is usually considerably lower and the fixed cost does not alter the analysis. Thus, while it is possible that a counterfeit drives out an authentic product by aggressive pricing, our model excludes the possibility that a brand name company eliminates a counterfeit from the market through pricing, which is true in most real cases.

It is easy to show that, if $c_a < \frac{2q_a(q_a - q_f)}{2q_a - q_f}$, the unique Nash equilibrium (p_a^*, p_f^*) is given by

$$p_a^* = \frac{2q_a(q_a - q_f + c_a)}{4q_a - q_f},$$
$$p_f^* = \frac{q_f(q_a - q_f + c_a)}{4q_a - q_f}.$$

Otherwise, the authentic product will be driven out of the market due to its high production cost and/or low quality. Furthermore, $p_a^* = \frac{2q_a p_f^*}{q_f} > 2p_f^*$, which holds for most products with non-deceptive counterfeits such as DVDs, CDs, luxury apparels and handbags. This is partly due to considerably lower variable cost, assumed zero in our model, of a counterfeit.

Throughout the paper, we will use the superscript "$*$" to represent equilibrium or optimal values, sometimes without the arguments. We show that a counterfeit lowers the authentic product's price and profit in the following proposition. All the proofs in the paper can be found in the Appendix.

Proposition 1 *The equilibrium price p_a^* and profit π_a^* are both decreasing in q_f. However, the total market share of the product, $P_a^* + P_f^*$, is increasing in q_f.*

The first result in Proposition 1 is not surprising. What is surprising is the extent a counterfeit may damage a brand name product. We conduct a numerical experiment to examine the pricing strategy of DVDs in the Chinese market where counterfeit DVDs are sold at around \$1. We estimate that $c_a = \$0.5$ and vary the quality level of an authentic DVD, q_a, in dollars between [\$15, \$25] which are around two to three times of a movie ticket. For a given q_a, we vary q_f in $[q_a - \$5, q_a - \$1]$ such that $q_f < q_a$ but keep $q_a - q_f$ small enough as the quality of a counterfeit DVD is quite close to the authentic one nowadays. With all the possible combinations we tested, the optimal price p_a^* is always less than \$3.5 with the average being \$2.33. This is consistent with Time Warner's pricing strategy in the Chinese market in which it has lowered the price for newly released DVDs to as low as \$2 to \$3 solely due to counterfeiting (Kelly 2005). An illustrative example is shown in Fig. 1.

While movie studios suffer great losses from counterfeiting DVDs, many luxury product manufacturers are able to maintain high prices even in markets with wide spread counterfeiting. We also conduct a numerical experiment for the pricing strategy of luxury bags (e.g., Louis Vuitton or Gucci). In this category, the quality difference can be huge. If we set q_a

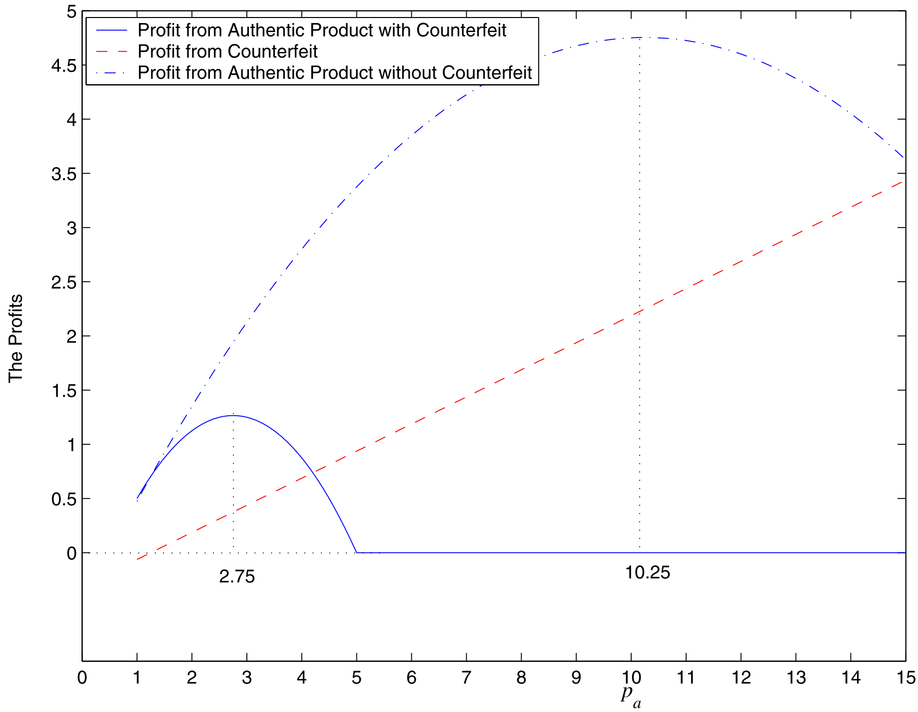

Fig. 1 Profits as functions of p_a for authentic and counterfeit DVDs at $q_a = \$20$, $q_f = \$16$, $c_a = \$0.5$ and $p_f = \$1$

between [\$1600, \$2100], c_a between [\$60, \$120] and q_f between [\$100, \$500], which are reasonable based on our experience, then the optimal price p_a^* is higher than \$600 and the average price is \$812 in over 99.5% of the combinations we tested. An illustrative example is shown in Fig. 2. As one can see, the impact of a quality counterfeit on the price of an authentic product is not as significant as that with DVDs due to significant quality advantage including brand values of luxury products.

One interesting result is that, *more* consumers will make a purchase, authentic or counterfeit, in the presence of a counterfeit due to a lower price of the brand name product and a low price counterfeit. Furthermore, as consumers with experience in a counterfeit may be more likely to try the authentic product, non-deceptive counterfeits may enhance the popularity of the authentic products and have promotional effect on the authentic products (Ritson 2007).

To protect itself from counterfeiting, a brand name company can exert effort to improve the quality of its own product. Such effort may result in a higher variable cost c_a (e.g., use better materials and implement better quality control techniques to improve the quality level of the authentic product) or require a fixed investment (e.g., initiate R&D projects or launch heavy advertisement on its products). Qian (2006) shows that a brand name company can improve the quality and profitability of its product if the counterfeits' quality is below a certain level and the incremental variable cost is not too high. Thus, we will study how a fixed investment can enhance the competitiveness of a brand name product.

A brand name company can also exert effort to reduce the quality level of a counterfeit. For instance, it can launch a marketing campaign to raise consumers' awareness of intellectual properties and the potential harm of counterfeits, or push the government for

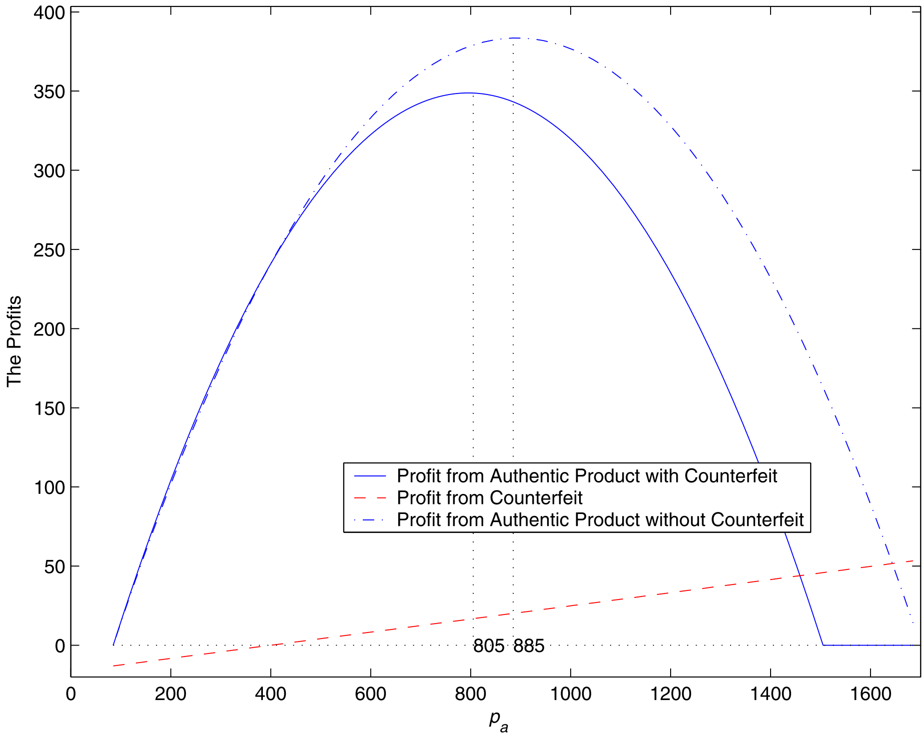

Fig. 2 Profits as functions of p_a for authentic and counterfeit handbags at $q_a = \$1,700$, $q_f = \$250$, $c_a = \$85$ and $p_f = \$60$

enforcement. Some French and Italian brand name companies of luxury goods are able to lobby their own governments to confiscate any counterfeit found at the customs from its user (Corbet 2005).

Suppose that, by making a fixed investment ξ, a brand name company will increase the quality of the authentic product to $q_a(\xi)$ (referred to as the quality improvement strategy) or decrease the perceived quality of the counterfeit to $q_f(\xi)$ (referred to as the direct fighting strategy). Let $\pi_a^{I*}(\xi) = \pi_a^I(p_a^*(\xi), p_f^*(\xi))$ $(\pi_a^{F*}(\xi) = \pi_a^F(p_a^*(\xi), p_f^*(\xi)))$ be the equilibrium profit of the brand name product with an investment of ξ and ξ^{I*} (ξ^{F*}) be the optimal investment level if the quality improvement (direct fighting strategy) is adopted. Then, for a given investment ξ, the brand name company will enhance the quality of its own product if and only if $\pi_a^{I*}(\xi) \geq \pi_a^{F*}(\xi)$. We show that, if the effectiveness of an investment on the quality levels under both strategies is the same, quality improvement is preferred at any investment level, and it requires less investment while achieving a higher profit.

Proposition 2 *Suppose that* $q_a(\xi) - q_a = q_f - q_f(\xi)$ *for all* ξ. *Then*

1. $\pi_a^{I*}(\xi) \geq \pi_a^{F*}(\xi)$ *and*
2. $\xi^{I*} \leq \xi^{F*}$ *and* $\pi_a^{I*}(\xi^{I*}) \geq \pi_a^{F*}(\xi^{F*})$.

That is, unless direct fighting is significantly more effective, companies should try to focus on improving the quality of its product. This explains why brand name companies are in general reluctant to fight counterfeits directly and, as an example, many counterfeiting products are sold openly in some streets in Hong Kong which are well known tourist attractions. A brand name company will take on its counterfeit directly only if it becomes too expensive to further improve the quality of their own products or they have very effective means for fighting counterfeiting directly.

3 Impact of counterfeiting in competitive markets

In many markets, there are multiple competing brand name products, e.g., shoes by Nike and Adidas and shirts by Ralph Lauren and Tommy Hilfiger. To consumers, these brand name products differ mostly in their brand values rather than their real quality levels, and their variable costs are almost identical. In this section, we consider a market with two competing brand name products a_1 and a_2, and assume that $q_{a_1} > q_{a_2}$ and $c_{a_1} = c_{a_2} = c$. There is a counterfeiter in the market who manufactures the counterfeit of product a_1 or a_2, or both. We assume that the quality levels of the counterfeits are lower than that of either authentic product. Again, we ignore their production costs as in Sect. 2. If the counterfeits have the same quality level, then their equilibrium prices must be the same and can be regarded as a single product. Otherwise, it is not difficult to show that it is more profitable for the counterfeiter to manufacture only the counterfeit with a higher quality level. Thus, we only need to consider one counterfeit, referred to as product f. The counterfeit will affect the brand name product with a lower quality because it will draw some consumers away from *both* brand name products.

For any given (p_{a_1}, p_{a_2}, p_f) satisfying $0 < \frac{p_f}{q_f} < \frac{p_{a_2}-p_f}{q_{a_2}-q_f} < \frac{p_{a_1}-p_{a_2}}{q_{a_1}-q_{a_2}} < 1$, conditions required for all three products to exist in the market, the market shares are as follows.

$$P_{a_1}(p_{a_1}, p_{a_2}, p_f) = 1 - \frac{p_{a_1} - p_{a_2}}{q_{a_1} - q_{a_2}},$$

$$P_{a_2}(p_{a_1}, p_{a_2}, p_f) = \frac{p_{a_1} - p_{a_2}}{q_{a_1} - q_{a_2}} - \frac{p_{a_2} - p_f}{q_{a_2} - q_f},$$

$$P_f(p_{a_1}, p_{a_2}, p_f) = \frac{p_{a_2} - p_f}{q_{a_2} - q_f} - \frac{p_f}{q_f}.$$

By solving the optimality equations $\frac{\partial_{p_i} \pi_i(p_{a_1}, p_{a_2}, p_f)}{\partial p_i} = 0$, $i = a_1, a_2, f$, we obtain the unique Nash equilibrium prices as

$$p^*_{a_1} = \frac{(6q_{a_1}q_{a_2} - 5q_{a_2}q_f - q_{a_1}q_f)c + (q_{a_1} - q_{a_2})(4q_{a_1}q_{a_2} - q_{a_1}q_f - 3q_{a_2}q_f)}{2(4q_{a_1}q_{a_2} - 2q_{a_2}q_f - q_{a_2}^2 - q_{a_1}q_f)},$$

$$p^*_{a_2} = \frac{q_{a_2}[(2q_{a_1} + q_{a_2} - 3q_f)c + (q_{a_1} - q_{a_2})(q_{a_2} - q_f)]}{4q_{a_1}q_{a_2} - 2q_{a_2}q_f - q_{a_2}^2 - q_{a_1}q_f},$$

$$p^*_f = \frac{q_f[(2q_{a_1} + q_{a_2} - 3q_f)c + (q_{a_1} - q_{a_2})(q_{a_2} - q_f)]}{2(4q_{a_1}q_{a_2} - 2q_{a_2}q_f - q_{a_2}^2 - q_{a_1}q_f)},$$

if $c < \frac{q_{a_2}(q_{a_2}-q_f)}{2q_{a_2}-q_f}$. Otherwise, product a_2 will be driven out of the market. It is easy to verify that $p^*_{a_1} > p^*_{a_2}$, $P^*_{a_1} > P^*_{a_2}$ and we refer to product a_1 as the big brand and a_2 as the small brand. Furthermore, we have $p^*_{a_2} > 2p^*_f$, similar to the previous result for a single product and its counterfeit due to low cost of a counterfeit.

Let $\bar{\pi}_i^*$, $i = a_1, a_2$, denote the profits of the brand name products in a competitive market absent of a counterfeit. The next proposition summarizes the impact of the counterfeit on the profitability of the two brand name products.

Proposition 3 *Although the absolute loss is higher for the big brand, the small brand suffers a higher relative loss, i.e.,* $\bar{\pi}_{a_1} - \pi_{a_1}^* > \bar{\pi}_{a_2} - \pi_{a_2}^*$ *and* $(\bar{\pi}_{a_2} - \pi_{a_2}^*)/\bar{\pi}_{a_2} > (\bar{\pi}_{a_1} - \pi_{a_1}^*)/\bar{\pi}_{a_1}$.

Proposition 3 reveals that a counterfeit of the big brand can be relatively more destructive to a small brand and may potentially enhance the competitiveness of a big brand. We observe this phenomenon in the Chinese market of office software systems where there were two major brand name products ten years ago, Microsoft Office as the big one and Kingsoft WPS Office as the small one. Kingsoft was a local software company once aspired to be "the Microsoft of China". Because of rampant counterfeiting of Microsoft Office, Kingsoft was not able to make enough profit in China. Eventually, it stopped selling the product and made it free for downloading from its web site. According to the vice president of the Business Software Alliance, a US based industry group, "When the piracy rate is as high as it is, it's hard for (Chinese) producers to develop a market, while the foreign developers have the whole world market" (Associated Press 2006).

Again, a brand name company can also invest in quality improvement or direct fighting to protect itself from counterfeiting. Note that, direct fighting reduces the quality level of a counterfeit and benefits not only the company itself but also its competitor. Thus, intuitively, the brand name companies are even *more* reluctant to fight counterfeiting directly *under competition*. While it is true for the big brand, it may not be true for the small brand. If q_{a_1} and q_{a_2} are close enough but the small brand is unable to overtake the big brand as the market leader with its investment, increasing the quality level of product a_2 may intensify the competition with the big brand and lower its own profit margin and hence profit. In such a case, it may be of the best interest for the small company to invest in reducing the quality of the counterfeit.

Now, we examine the role each brand name company plays if direct fighting is the only option, which is true if it is too difficult for both companies to improve the quality of their products further. Let ξ_1 and ξ_2 be the efforts exerted by brand name companies a_1 and a_2, and $\xi = r_1\xi_1 + r_2\xi_2$ be the total effective fighting effort where $(r_1, r_2) > 0$ measure the effectiveness of direct fighting by the brand name companies. Let $q_f(\xi)$ be the perceived quality level of the counterfeit and $\pi_i^*(\xi) = \pi_i^*(p_{a_1}^*(\xi), p_{a_2}^*(\xi), p_f^*(\xi))$, $i = a_1, a_2$, for notational simplicity as functions of the investment levels ξ. By Lemma 1, $\pi_{a_2}^*(\xi)$ is concave if it has no inflection point,[1] and first convex and then concave as ξ increases if it has an inflection point, while $\pi_{a_1}^*(\xi)$ is always concave.

Lemma 1 *Suppose that* $q_f(\xi)$ *is decreasing convex in* ξ *with* $q_f'''(\xi)q_f'(\xi) \le 2[q_f''(\xi)]^2$. *Then,* $\pi_{a_1}^*(\xi)$ *is strictly concave in* ξ, *and* $\pi_{a_2}^*(\xi)$ *has at most one inflection point. If* $\lim_{\xi\to+\infty} q_f'(\xi) = 0$, $\pi_{a_2}^*(\xi)$ *is concave when* ξ *is large enough. A sufficient condition for* $\pi_{a_2}^*(\xi)$ *to be strictly concave is* $c = 0$.

The conditions in Lemma 1 imply diminishing marginal effectiveness of the fighting effort ξ on the quality of the counterfeit, which eventually becomes zero as ξ approaches

[1] An inflection point is a point at which the convexity of a function changes its sign, e.g., from convex to concave, or from concave to convex.

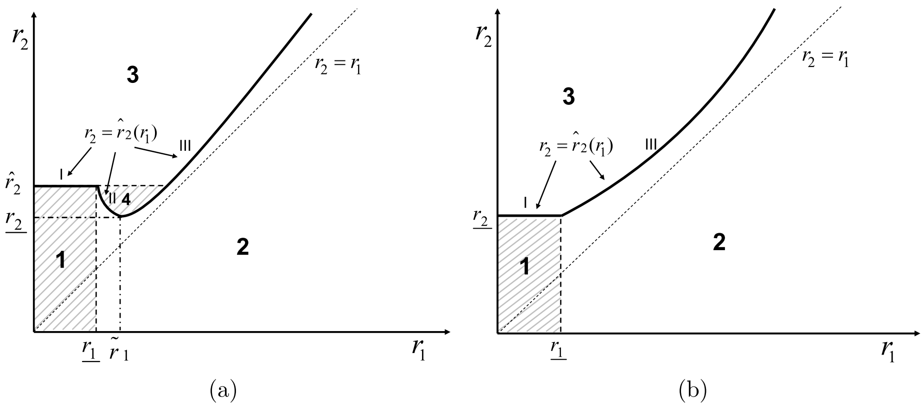

Fig. 3 Equilibrium fighting strategies at two competing brand name companies

infinite. Furthermore, $q_f'''(\xi)q_f'(\xi) \le 2[q_f''(\xi)]^2$ is met by many functions including the exponential function $q_f(\xi) = Ae^{-k\xi} + B$ and power function $q_f(\xi) = A(\xi + C)^{-k} + B$ where $A, B, C, k > 0$.

The next proposition summarizes the optimal investment decisions at the brand name companies. The detailed mathematical expressions of the parameters in the proposition and the proof are provided in Sect. A.5.

Proposition 4 *Under the conditions in Lemma 1, there exists a unique function $\hat{r}_2(r_1) > r_1$, with which we can then characterize the equilibrium investments by the two brand name companies, (ξ_1^*, ξ_2^*). If $\pi_{a_2}^*(\xi)$ is first convex and then concave, (ξ_1^*, ξ_2^*) can be characterized as follows and shown in Fig. 3(a).*

1. *When (r_1, r_2) falls in Region 1, i.e., $0 \le r_1 \le \underline{r_1}$ and $r_2 \le \hat{r}_2(r_1)$, there exists a unique Nash equilibrium under which no company will fight, i.e., $\xi_1^* = \xi_2^* = 0$.*
2. *When (r_1, r_2) falls in Region 2, i.e., $r_2 \le \hat{r}_2(r_1)$ if $\underline{r_1} < r_1 \le \tilde{r}_1$ and $r_2 < \hat{r}_2(r_1)$ if $r_1 > \tilde{r}_1$, there exists a unique Nash equilibrium under which only company a_1 will fight, i.e., $\xi_1^* > 0$ and $\xi_2^* = 0$.*
3. *When (r_1, r_2) falls in Region 3, i.e., $r_2 > \max\{\hat{r}_2, \hat{r}_2(r_1)\}$, there exists a unique Nash equilibrium under which only company a_2 will fight, i.e., $\xi_1^* = 0$ and $\xi_2^* > 0$.*
4. *When (r_1, r_2) falls in Region 4, i.e., $r_1 > \underline{r_1}$ and $\hat{r}_2(r_1) < r_2 \le \hat{r}_2$, there does not exist any Nash equilibrium.*[2]
5. *When $r_1 > \tilde{r}_1$ and $r_2 = \hat{r}_2(r_1)$, i.e., (r_1, r_2) is on III, there are multiple equilibria, some of which result in joint effort in fighting counterfeits by both companies.*

If $\pi_{a_2}^*(\xi)$ is concave, Region 4 no longer exists as shown in Fig. 3(b).

Note that, when $\pi_{a_2}^*(\xi)$ is first convex and then concave, the function $r_2 = \hat{r}_2(r_1)$ consists of three segments, denoted as I (when $r_1 \le \underline{r_1}$), II (when $\underline{r_1} < r_1 < \tilde{r}_1$) and III (when $r_1 \ge \tilde{r}_1$)

[2] In this region, company a_1 will fight if company a_2 does not. When company a_1 fights, company a_2 also has an incentive to add more investment to reach its optimal level. However, when company a_2 fights, company a_1 has no more incentive to fight since it can rely on company a_2's effort. When company a_1 does not fight, company a_2 will not fight either. Then company a_1 has an incentive to fight again …

in Fig. 3(a). On segment I, $\hat{r}_2(r_1) = \hat{r}_2$ is a constant. Segment II is associated with the convex part of $\pi^*_{a_2}(\xi)$, while segment III is associated with the concave part of $\pi^*_{a_2}(\xi)$. They join at $(\tilde{r}_1, \underline{r_2})$ which is the minimum of $\hat{r}_2(r_1)$. Hence, segment II does not exist when $\pi^*_{a_2}(\xi)$ is concave for all $\xi \geq 0$.

As we can see, the size and fighting power of the brand name companies jointly determine who will take on the responsibility in direct fighting. A small brand will simply rely on a big brand to fight counterfeiting unless it is much more effective in fighting (i.e., $r_2 > \hat{r}_2(r_1) > r_1$). This may partly explain why most local sports shoe makers in China never bother to fight counterfeiting since their foreign competitors such as Nike and Adidas have much larger market shares and higher fighting power. On the other hand, if a small local brand is more effective in its effort, e.g., it has close ties with the government, it should take the initiative.

It is interesting to observe that, when (r_1, r_2) falls into Region 4 in Fig. 3(a), it calls for cooperation between the brand name companies in devising their fighting strategies. This is because, although the small brand has higher fighting power when $r_2 > \hat{r}_2(r_1) > r_1$, it is too costly for it to fight alone. In this case, cooperation is a better choice and both companies are better off by joining forces. The Recording Industry Association of America (RIAA) and the Motion Picture Association of America (MPAA) are successful examples in fighting global counterfeiting.

4 Conclusion

Although counterfeits are widely available in many markets and causing severe damage to both brand name products and consumers, there has been little analytical study on the impact of counterfeits on brand name products and the strategies for brand name companies to protect their brand names and fight counterfeiting. In this paper, we adopt a vertical differentiation model to analyze the impact of non-deceptive counterfeits on brand name products.

We first consider a market with a brand name product and a non-deceptive counterfeit. As expected, the counterfeit lowers the price and profit of the authentic product. To see the magnitudes of the impact, we conduct numerical studies on the equilibrium prices for DVDs and luxury handbags using data collected in the Chinese markets. These studies support distinct pricing strategies adopted by the movie studios and luxury manufacturers in the Chinese market. Furthermore, the impact of a counterfeit on the profit of a brand name product increases as the quality difference between the authentic product and its counterfeit narrows. On the other hand, a counterfeit drives more consumers to make purchases and hence may have promotional effect on and enhance the popularity of an authentic product. This is consistent with the findings by Ritson (2007) that some brand name companies are willing to sacrifice short term profits for larger market shares in the future.

A brand name company can exert some effort to fight counterfeiting by either improving the overall quality of its own product, or reduce that of the counterfeit. We show that, a brand name company is more likely to invest in improving its own product's quality than reducing that the counterfeit's perceived quality, if the two strategies are equally effective in changing the quality levels. That is, a brand name company will take on the counterfeit directly only if it is much more effective to do so. Furthermore, improving its own quality level requires less effort. These results explain why so many brand name companies are reluctant to fight counterfeiting directly.

We then extend this model to a market with two competing brand name products and a counterfeit. To the best of our knowledge, we are the first to analyze the impact of counterfeiting and the fighting strategies in a competitive market. In such a market, a counterfeit of one brand name product can draw some consumers away from and damage the profitability of the other brand name product. Although the absolute impact of the counterfeit on the big brand is higher, the relative impact is lower than the small brand. Thus, both the brand name companies are victims of the counterfeit and have incentive to fight counterfeiting. With competition, intuitively, the brand name companies are even more reluctant to fight counterfeiting directly. However, increasing the quality of the small brand name product may intensify the competition between the two brand name products and it may be of the best interest of the small brand to fight the counterfeit directly.

If direct fighting is the only option, we derive the equilibrium fighting strategies and show that the small brand will simply rely on the big brand's fighting effort unless it is more effective than the big brand in fighting counterfeiting. Under certain conditions, there exist no or multiple Nash equilibria, which calls for cooperation between the two brand name companies as none of them is powerful enough to fight counterfeiting alone.

Acknowledgement The authors would like to thank the editor and two anonymous referees for their helpful comments and suggestions. This work was supported in part by the Hong Kong Research Grants Council under grants 621705, 622406 and 622608.

Appendix: Proofs

A.1 Proof of Proposition 1

Proof From the equilibrium prices we obtain, the market share of the brand name company is

$$P_a^* = \frac{2q_a(q_a - q_f) - c_a(2q_a - q_f)}{(q_a - q_f)(4q_a - q_f)} = \frac{2q_a^2 - 2q_aq_f - 2q_ac_a + q_fc_a}{(q_a - q_f)(4q_a - q_f)}.$$

Note that, $P_a^* > 0$ implies that $c_a < \frac{2q_a(q_a - q_f)}{2q_a - q_f}$. Therefore,

$$\frac{\partial p_a^*}{\partial q_f} = -\frac{2q_a(3q_a - c_a)}{(4q_a - q_f)^2} < 0,$$

$$\frac{\partial \pi_a^*}{\partial q_f} = -\frac{(2q_a^2 - 2q_aq_f - 2q_ac_a + c_aq_f)(4q_a^3 - 2q_a^2q_f - 2q_aq_f^2 + 4q_a^2c_a - 2q_aq_fc_a + c_aq_f^2)}{(4q_a - q_f)^3(q_a - q_f)^2}$$
$$< 0.$$

Therefore, p_a^* and π_a^* are decreasing in q_f. Since $P_a^* + P_f^* = 1 - p_f^*/q_f = 1 - p_a^*/2q_a$, $P_a^* + P_f^*$ is increasing in q_f. □

A.2 Proof of Proposition 2

Proof We prove the properties in two steps.

1. Because $q_a(\xi) - q_a = q_f - q_f(\xi)$, $q_a(\xi) - q_f = q_a - q_f(\xi)$. Let $q_a(\xi) - q_f = q_a - q_f(\xi) = \delta$. Then the profit of the brand name manufacturer when the direct fighting strategy is adopted is:

$$\pi_a^{F*}(\xi) = \frac{[2q_a^2 - 2q_a q_f(\xi) - 2q_a c_a + q_f(\xi)c_a]^2}{[4q_a - q_f(\xi)][q_a - q_f(\xi)]}$$
$$= \frac{(2q_a\delta - c_a\delta - q_a c_a)^2}{\delta(3q_a + \delta)},$$

and the profit for the quality improvement strategy is $\pi_a^{I*}(\xi) = \frac{[2q_a(\xi)\delta - c_a\delta - q_a(\xi)c_a]^2}{\delta[3q_a(\xi)+\delta]}$. Note that, $P_a^{F*}(\xi) > 0$ implies that $\delta > \frac{q_a c_a}{2q_a - c_a}$. Since

$$\frac{\partial}{\partial q_a}\left[\frac{(2q_a\delta - c_a\delta - q_a c_a)^2}{\delta(3q_a + \delta)}\right] = \frac{(2q_a\delta - c_a\delta - q_a c_a)[4\delta^2 + (6q_a + c_a)\delta - 3q_a c_a]}{\delta(3q_a + \delta)^2}$$

and $4\delta^2 + (6q_a + c_a)\delta - 3q_a c_a > 0$ for $\delta > \frac{q_a c_a}{2q_a - c_a}$, $\pi_a^{I*}(\xi) > \pi_a^{F*}(\xi)$ as $q_a(\xi) > q_a$.

2. It is obvious that $\pi_a^{I*}(\xi^{I*}) = \max_\xi \pi_a^{I*}(\xi) > \pi_a^{F*}(\xi^{F*}) = \max_\xi \pi_a^{I*}(\xi)$ from the previous result. Next, we show that $\xi^{I*} \le \xi^{F*}$ by proving $\pi_a^{I*'}(\xi) \ge \pi_a^{F*'}(\xi)$.

   Since $P_a^* > 0$ implies that $c_a < \frac{2q_a(q_a - q_f)}{2q_a - q_f}$, we have

$$\frac{\partial\pi_a^*}{\partial q_a} + \frac{\partial\pi_a^*}{\partial q_f} = \frac{4(q_a - q_f + c_a)(2q_a^2 - 2q_a q_f - 2q_a c_a + c_a q_f)}{(4q_a - q_f)^3} \ge 0.$$

   Since $q_a(\xi) - q_a = q_f - q_f(\xi)$ for any $\xi \ge 0$, $q_a'(\xi) = -q_f'(\xi)$. Therefore, we have $\pi_a^{I*'}(\xi) = q_a'(\xi)\frac{\partial\pi_a^*}{\partial q_a} \ge q_f'(\xi)\frac{\partial\pi_a^*}{\partial q_f} = \pi_a^{F*'}(\xi)$.

□

A.3 Proof of Proposition 3

Proof Note that, $P_i^* > 0$, $i = a_1, a_2, f$, if and only if

$$c < \frac{q_{a_2}(q_{a_2} - q_f)}{2q_{a_2} - q_f}. \tag{1}$$

We can show that

$$\frac{\partial(\pi_{a_1}^* - \pi_{a_2}^*)}{\partial q_f} = -\frac{(q_{a_1} - q_{a_2})F(q_{a_1}, q_{a_2}, q_f)}{2(q_{a_2} - q_f)^2(4q_{a_1}q_{a_2} - q_{a_1}q_f - 2q_{a_2}q_f - q_{a_2}^2)^3} < 0,$$
$$\frac{\partial(\pi_{a_1}^*/\pi_{a_2}^*)}{\partial q_f} = \frac{G(q_{a_1}, q_{a_2}, q_f)P_{a_1}^*/P_{a_2}^*}{2(q_{a_1} - q_f)(q_{a_2} - q_f)[q_{a_2}(q_{a_2} - q_f) - (2q_{a_2} - q_f)c]^2} > 0,$$

because $F(q_{a_1}, q_{a_2}, q_f) > 0$ and $G(q_{a_1}, q_{a_2}, q_f) > 0$ when (1) holds (see the online companion) and $4q_{a_1}q_{a_2} - q_{a_1}q_f - 2q_{a_2}q_f - q_{a_2}^2 > 0$ as $q_{a_1} > q_{a_2} > q_f$. Therefore, $\pi_{a_1}^* - \pi_{a_2}^*$ decreases in q_f and $\pi_{a_1}^*/\pi_{a_2}^*$ increases in q_f. Note that, $\bar{\pi}_{a_i}$, $i = 1, 2$ are equal to π_{a_i} when $q_f = 0$. Thus, $\bar{\pi}_{a_1} - \pi_{a_1}^* > \bar{\pi}_{a_2} - \pi_{a_2}^*$ and $\pi_{a_1}^*/\pi_{a_2}^* > \bar{\pi}_{a_1}^*/\bar{\pi}_{a_2}^*$, which implies that $(\bar{\pi}_{a_2} - \pi_{a_2}^*)/\bar{\pi}_{a_2} > (\bar{\pi}_{a_1} - \pi_{a_1}^*)/\bar{\pi}_{a_1}$. □

A.4 Proof of Lemma 1

Proof Note that, for $i = 1, 2$,

$$\pi_{a_i}^{*'}(\xi) = q_f'(\xi)\partial_{q_f}\pi_{a_i}^*,$$

$$\pi_{a_i}^{*''}(\xi) = q_f''(\xi)\partial_{q_f}\pi_{a_i}^* + [q_f'(\xi)]^2\frac{\partial^2\pi_{a_i}^*}{\partial q_f^2},$$

and $q_f'(\xi) < 0$, $q_f''(\xi) > 0$. We have

$$\frac{\partial\pi_{a_1}^*}{\partial q_f} = -\frac{q_{a_2}(q_{a_1}-q_{a_2})[(3q_{a_2}-2c)(q_{a_1}-q_{a_2})+3q_{a_2}c]P_{a_1}^*}{(4q_{a_1}q_{a_2}-q_{a_1}q_f-2q_{a_2}q_f-q_{a_2}^2)^2} < 0,$$

$$\frac{\partial^2\pi_{a_1}^*}{\partial q_f^2} = \frac{q_{a_2}(q_{a_1}-q_{a_2})[(3q_{a_2}-2c)(q_{a_1}-q_{a_2})+3q_{a_2}c]H(q_{a_1},q_{a_2},q_f)}{2(4q_{a_1}q_{a_2}-q_{a_1}q_f-2q_{a_2}q_f-q_{a_2}^2)^4},$$

where

$$\begin{aligned}
&H(q_{a_1},q_{a_2},q_f)\\
&\quad = (2q_{a_1}q_{a_2}-2q_{a_1}q_f+13q_{a_2}^2-4q_{a_2}q_f)c+2q_{a_1}^2q_f-8q_{a_2}q_{a_1}^2+10q_{a_1}q_{a_2}q_f-13q_{a_2}^2q_{a_1}\\
&\qquad -3q_{a_2}^3+12q_{a_2}^2q_f\\
&\quad < (2q_{a_1}q_{a_2}-2q_{a_1}q_f+13q_{a_2}^2-4q_{a_2}q_f)\frac{q_{a_2}(q_{a_2}-q_f)}{2q_{a_2}-q_f}+2q_{a_1}^2q_f-8q_{a_2}q_{a_1}^2+10q_{a_1}q_{a_2}q_f\\
&\qquad -13q_{a_2}^2q_{a_1}-3q_{a_2}^3+12q_{a_2}^2q_f\\
&\quad = -(4q_{a_1}q_{a_2}-2q_{a_2}q_f-q_{a_2}^2-q_{a_1}q_f)(7q_{a_2}^2-4q_{a_2}q_f+4q_{a_1}q_{a_2}-2q_{a_1}q_f)/(2q_{a_2}-q_f)\\
&\quad < 0.
\end{aligned}$$

Thus, $\pi_{a_1}^{*'}(\xi) > 0$ and $\pi_{a_1}^{*''}(\xi) < 0$. So $\pi_{a_1}^*(\xi)$ is strictly concave.

Next, we study the monotonicity and concavity of $\pi_{a_2}^*(\xi)$. We have

$$\frac{\partial\pi_{a_2}^*}{\partial q_f} = -\frac{(q_{a_1}-q_{a_2})[q_{a_2}(q_{a_2}-q_f)-(2q_{a_2}-q_f)c]J_1(q_{a_1},q_{a_2},q_f)}{(q_{a_2}-q_f)^2(4q_{a_1}q_{a_2}-q_{a_1}q_f-2q_{a_2}q_f-q_{a_2}^2)^3} < 0$$

because $J_1(q_{a_1},q_{a_2},q_f) > 0$ (see the online companion) and condition (1), we have $\pi_{a_2}^{*'}(\xi) > 0$ and $\pi_{a_2}^*(\xi)$ is increasing.

To show that $\pi_{a_2}^*(\xi)$ has at most one inflection point, we only need to establish that once the function becomes concave as ξ increases, it will always be concave. Note that concavity of $\pi_{a_2}^*(\xi)$ means

$$[q_f'(\xi)]^2\frac{\partial^2\pi_{a_2}^*}{\partial q_f^2} < -q_f''(\xi)\frac{\partial\pi_{a_2}^*}{\partial q_f},$$

or

$$-\frac{\frac{\partial^2\pi_{a_2}^*}{\partial q_f^2}}{\frac{\partial\pi_{a_2}^*}{\partial q_f}}\frac{[q_f'(\xi)]^2}{q_f''(\xi)} < 1 \tag{2}$$

which holds if $\frac{\partial^2\pi_{a_2}^*}{\partial q_f^2} \le 0$. Thus, it is sufficient to show that the left hand side of (2) is decreasing in ξ if $\frac{\partial^2\pi_{a_2}^*}{\partial q_f^2} > 0$. Because $q_f'''(\xi)q_f'(\xi) \le 2[q_f''(\xi)]^2$, it is easy to show that $\frac{[q_f'(\xi)]^2}{q_f''(\xi)}$

is decreasing in ξ, we only need to show that $-\frac{\partial^2\pi^*_{a_2}}{\partial q_f^2}/\frac{\partial \pi^*_{a_2}}{\partial q_f}$ is decreasing in ξ if $\frac{\partial^2\pi^*_{a_2}}{\partial q_f^2} > 0$. A sufficient condition for this to be true is $\frac{\partial^3\pi^*_{a_2}}{\partial q_f^3} > 0$ if $\frac{\partial^2\pi^*_{a_2}}{\partial q_f^2} > 0$. Since

$$\frac{\partial^2\pi^*_{a_2}}{\partial q_f^2} = \frac{2(q_{a_1} - q_{a_2})J_2(q_{a_1}, q_{a_2}, q_f)}{(q_{a_2} - q_f)^3(4q_{a_1}q_{a_2} - q_{a_1}q_f - 2q_{a_2}q_f - q_{a_2}^2)^4}$$

where $J_2(q_{a_1}, q_{a_2}, q_f)$ is a complicated polynomial, $\frac{\partial^2\pi^*_{a_2}}{\partial q_f^2} > 0$ if and only if $J_2(q_{a_1}, q_{a_2}, q_f) > 0$. When $c = 0$, we show that $J_2(q_{a_1}, q_{a_2}, q_f) < 0$ in the online companion and $\pi^*_{a_2}(\xi)$ is strictly concave in ξ. Then, we only need to consider $c > 0$. Consider the third order derivative

$$\frac{\partial^3\pi^*_{a_2}}{\partial q_f^3} = \frac{2(q_{a_1} - q_{a_2})J_3(q_{a_1}, q_{a_2}, q_f)}{(q_{a_2} - q_f)^4(4q_{a_1}q_{a_2} - q_{a_1}q_f - 2q_{a_2}q_f - q_{a_2}^2)^4} + \frac{8(q_{a_1} - q_{a_2})(q_{a_1} + 2q_{a_2})J_2(q_{a_1}, q_{a_2}, q_f)}{(q_{a_2} - q_f)^3(4q_{a_1}q_{a_2} - q_{a_1}q_f - 2q_{a_2}q_f - q_{a_2}^2)^5}$$

where $J_3(q_{a_1}, q_{a_2}, q_f) = (q_{a_2} - q_f)\partial_{q_f} J_2(q_{a_1}, q_{a_2}, q_f) + 3J_2(q_{a_1}, q_{a_2}, q_f)$. We show in the online companion that $J_3(q_{a_1}, q_{a_2}, q_f) > 0$ if $J_2(q_{a_1}, q_{a_2}, q_f) > 0$. Therefore, $\frac{\partial^3\pi^*_{a_2}}{\partial q_f^3} > 0$ when $\frac{\partial^2\pi^*_{a_2}}{\partial q_f^2} > 0$.

Since $\lim_{\xi\to+\infty} q'_f(\xi) = 0$, $\lim_{\xi\to+\infty} \pi^{*'}_{a_2}(\xi) = 0$ and $\pi^*_{a_2}(\xi)$ is concave for large ξ values. □

A.5 Proof of Proposition 4

Proof Let $\Pi_1(\xi_1, \xi_2) = \pi^*_{a_1}(\xi) - \xi_1$ and $\Pi_2(\xi_1, \xi_2) = \pi^*_{a_2}(\xi) - \xi_2$ be the profits generated from products a_1 and a_2 after investing ξ_1 and ξ_2, respectively. Then, an equilibrium $\xi_i > 0$ must satisfy the best response equations $\frac{\partial \Pi_i(\xi_1,\xi_2)}{\partial \xi_i} = 0$ for $i = 1, 2$ or

$$r_i\pi^{*'}_{a_i}(\xi) - 1 = 0. \tag{3}$$

We first define several new parameters for characterizing the function $\hat{r}_2(r_1)$. Let $\tilde{\xi} = \arg\max_\xi\{\pi^{*'}_{a_2}(\xi)\}$, $\tilde{r}_1 = [\pi^{*'}_{a_1}(\tilde{\xi})]^{-1}$, and

$$\underline{r_1} = \left[\max_{\xi\ge 0}\{\pi^{*'}_{a_1}(\xi)\}\right]^{-1} = [\pi^{*'}_{a_1}(0)]^{-1},$$

$$\underline{r_2} = \left[\max_{\xi\ge 0}\{\pi^{*'}_{a_2}(\xi)\}\right]^{-1} = [\pi^{*'}_{a_2}(\tilde{\xi})]^{-1}.$$

Note that $\tilde{\xi}$ is the inflection point of $\pi^*_{a_2}(\xi)$ or zero. Since $\pi^*_{a_2}(\xi)$ has at most one inflection point and is concave when ξ is large enough by Lemma 1, the sufficient and necessary condition for $\pi^*_{a_2}(\xi)$ to be always concave is $\tilde{\xi} = 0$, in which case $\tilde{r}_1 = \underline{r_1}$. Otherwise, $\tilde{\xi} > 0$ and $\tilde{r}_1 > \underline{r_1}$.

Let $\hat{\xi}_1 = \text{argmax}_{\xi_1}\{\Pi_1(\xi_1, 0)\}$ be company a_1's optimal effort if company a_2 does not invest. If $\pi^*_{a_2}(\xi)$ has an inflection point, then $\pi^*_{a_2}(\xi)$ is concave only when $\xi \geq \tilde{\xi}$, and $\Pi_2(\hat{\xi}_1, \xi_2)$ may have two local maximizers: 0 and the solution of the equation $\frac{\partial \Pi_2(\hat{\xi}_1, \xi_2)}{\partial \xi_2} = 0$. However, when $r_2 \leq \underline{r_2}$, $\Pi_2(\hat{\xi}_1, \xi_2)$ has only one local maximizer which is zero, since $\frac{\partial \Pi_2(\hat{\xi}_1, \xi_2)}{\partial \xi_2} = r_2 \pi^{*'}_{a_2}(\xi) - 1 < 0$. Since $\pi^*_{a_1}(\xi)$ is concave and $\pi^{*'}_{a_1}(\xi)$ is decreasing, $r_1\hat{\xi}_1$ increases in r_1, where $\hat{\xi}_1$ is an implicit function of r_1. Thus, when $r_1 \geq \tilde{r}_1$, $r_1\hat{\xi}_1 \geq \tilde{\xi}$ and hence, $\Pi_2(\hat{\xi}_1, \xi_2) = \pi^*_{a_2}(r_1\hat{\xi}_1 + r_2\xi_2) - \xi_2$ is concave. In that case, $\Pi_2(\hat{\xi}_1, \xi_2)$ has only one local maximizer also. When $r_2 > \underline{r_2}$ and $r_1 < \tilde{r}_1$, $\Pi_2(\hat{\xi}_1, \xi_2)$ has exactly two local maximizers.

We then define the three segments of the function $\hat{r}_2(r_1)$, denoted as $\hat{r}_2^{\text{I}}(r_1)$, $\hat{r}_2^{\text{II}}(r_1)$ and $\hat{r}_2^{\text{III}}(r_1)$.

1. Let $\hat{r}_2^{\text{I}}(r_1)$ be the solution of the equation $\max_{\xi_2>0}\{\Pi_2(0, \xi_2)\} - \Pi_2(0, 0) = 0$ for $r_1 \leq \underline{r_1}$, if $\pi^*_{a_2}(\xi)$ has an inflection point. Note that $\hat{r}_2^{\text{I}}(r_1)$ is a constant (denoted as $\hat{r}_2$) and it is greater than $\underline{r_2}$. If $\pi^*_{a_2}(\xi)$ is concave for all ξ, let $\hat{r}_2^{\text{I}}(r_1) = \underline{r_2}$.
2. Let $\hat{r}_2^{\text{II}}(r_1)$ be the solution of the equation

$$\max_{\xi_2>0}\{\Pi_2(\hat{\xi}_1, \xi_2)\} - \Pi_2(\hat{\xi}_1, 0) = 0$$

for $r_1 \in (\underline{r_1}, \tilde{r}_1)$. When $r_2 = \hat{r}_2^{\text{II}}(r_1)$, $\Pi_2(\hat{\xi}_1, \xi_2)$ has exactly two *global* maximizers and hence, $\hat{r}_2^{\text{II}}(r_1) > \underline{r_2}$. If $\pi^*_{a_2}(\xi)$ is concave for all ξ, $\underline{r_1} = \tilde{r}_1$ and in that case, $\hat{r}_2^{\text{II}}(r_1)$ does not exist.

We can show that $\lim_{r_1\to\tilde{r}_1} \hat{r}_2^{\text{II}}(r_1) = \underline{r_2}$ and $\hat{r}_2^{\text{II}}(r_1) < \hat{r}_2$. Since $\hat{r}_2^{\text{II}}(r_1) > \underline{r_2}$ for any $r_1 < \tilde{r}_1$, $\lim_{r_1\to\tilde{r}_1} \hat{r}_2^{\text{II}}(r_1) \geq \underline{r_2}$. Then, we show that for any small enough ϵ, there exists $r_1 < \tilde{r}_1$ such that $\underline{r_2} < \hat{r}_2^{\text{II}}(r_1) < \underline{r_2} + \epsilon < \hat{r}_2$. Let $f(\xi_1) = \max_{\xi_2>0} \Pi_2(\xi_1, \xi_2) - \Pi_2(\xi_1, 0)$ for any given $r_2 \in (\underline{r_2}, \underline{r_2} + \epsilon)$. When $\xi_1 = 0$, $f(0) < 0$ since $r_2 < \hat{r}_2$; when $\xi_1 = \tilde{\xi}/r_1$, $f(\xi_1) > 0$ since $r_2 > \underline{r_2}$ and $\Pi_2(\xi_1, \xi_2)$ is concave in ξ_2. Therefore, there exists $\xi_1 \in (0, \tilde{\xi}/r_1)$ such that $f(\xi_1) = 0$. Furthermore, there exists $r_1 < \tilde{r}_1$ such that this ξ_1 also maximizes $\Pi_1(\xi_1, 0)$, i.e., $r_2 = \hat{r}_2^{\text{II}}(r_1)$, since $\pi^*_{a_1}(\xi)$ is concave and $\pi^{*'}_{a_1}(\xi)$ is decreasing. Hereby we complete the proof of $\lim_{r_1\to\tilde{r}_1} \hat{r}_2^{\text{II}}(r_1) = \underline{r_2}$.

We next prove $\hat{r}_2^{\text{II}}(r_1) < \hat{r}_2$. Suppose the contrary. Then, there exists $\underline{r_1} < r_1 < \tilde{r}_1$ such that $\hat{r}_2^{\text{II}}(r_1) = \hat{r}_2$, and then

$$\begin{aligned}
&\max_{\xi_2>0}\{\Pi_2(\hat{\xi}_1, \xi_2)\} - \Pi_2(\hat{\xi}_1, 0) \\
&\quad = \Pi_2\left(\hat{\xi}_1, \hat{\xi}_2 - \frac{r_1\hat{\xi}_1}{\hat{r}_2}\right) - \Pi_2(\hat{\xi}_1, 0) = \Pi_2(0, \hat{\xi}_2) + \frac{r_1\hat{\xi}_1}{\hat{r}_2} - \Pi_2(\hat{\xi}_1, 0) \\
&\quad = \max_{\xi_2>0}\{\Pi_2(0, \xi_2)\} + \frac{r_1\hat{\xi}_1}{\hat{r}_2} - \Pi_2(\hat{\xi}_1, 0) = \Pi_2(0, 0) + \frac{r_1\hat{\xi}_1}{\hat{r}_2} - \Pi_2(\hat{\xi}_1, 0) = 0,
\end{aligned}$$

where $\hat{\xi}_2 = \arg\max_{\xi_2>0}\{\Pi_2(0, \xi_2)\}$. Then, $\Pi_2(0, 0) = \Pi_2(\hat{\xi}_1, 0) - r_1\hat{\xi}_1/\hat{r}_2 = \Pi_2(0, r_1\hat{\xi}_1/\hat{r}_2)$. Recall that, when $r_2 = \hat{r}_2$, $\Pi_2(0, 0) = \max_{\xi_2>0}\{\Pi_2(0, \xi_2)\} = \Pi_2(0, \hat{\xi}_2)$. Thus, $\frac{r_1\hat{\xi}_1}{\hat{r}_2} = \hat{\xi}_2$. However, since $r_1 < \tilde{r}_1$ and $\pi^*_{a_2}(\xi)$ is concave when $\xi \geq \tilde{\xi}$, we have $r_1\hat{\xi}_1 < \tilde{\xi} < r_2\hat{\xi}_2$ which implies $\hat{\xi}_2 - \frac{r_1\hat{\xi}_1}{\hat{r}_2} > 0$, a contradiction. Therefore, $\hat{r}_2^{\text{II}}(r_1) < \hat{r}_2$.

3. Let $\hat{r}_2^{\mathrm{III}}(r_1) = [\pi_{a_2}^{*'}(r_1\hat{\xi}_1)]^{-1}$ for $r_1 \geq \tilde{r}_1$. When $r_2 = \hat{r}_2^{\mathrm{III}}(r_1)$, $r_1\hat{\xi}_1$ satisfies the two equations (3) simultaneously. When $r_2 > \hat{r}_2^{\mathrm{III}}(r_1)$,

$$\left.\frac{\partial \Pi_2(\hat{\xi}_1, \xi_2)}{\partial \xi_2}\right|_{\xi_2=0} = r_2\pi_{a_2}^{*'}(r_1\hat{\xi}_1) - 1 > \hat{r}_2^{\mathrm{III}}(r_1)\pi_{a_2}^{*'}(r_1\hat{\xi}_1) - 1 = 0.$$

Thus, $\max_{\xi_2\geq 0}\{\Pi_2(\hat{\xi}_1, \xi_2)\} > \Pi_2(\hat{\xi}_1, 0)$. Similarly, when $r_2 \leq \hat{r}_2^{\mathrm{III}}(r_1)$, $\max_{\xi_2\geq 0}\{\Pi_2(\hat{\xi}_1, \xi_2)\} = \Pi_2(\hat{\xi}_1, 0)$. Furthermore, when $r_1 = \tilde{r}_1$, $\hat{\xi}_1 = \tilde{\xi}/r_1$ and $\underline{r_2} = \hat{r}_2^{\mathrm{III}}(\tilde{r}_1)$. Since $r_1\hat{\xi}_1$ increases in r_1 and $\pi_{a_2}^{*'}(\xi)$ decreases in ξ when $\xi \geq \tilde{\xi}$, $\hat{r}_2^{\mathrm{III}}(r_1)$ increases in r_1. Therefore, $\hat{r}_2(r_1)$ is continuous and attains its minimum at $r_1 = \tilde{r}_1$.

From the above definition of the function $\hat{r}_2(r_1)$, it is not difficult to see that,

$$\max_{\xi_2\geq 0}\{\Pi_2(\hat{\xi}_1, \xi_2)\} \quad \begin{cases} > \Pi_2(\hat{\xi}_1, 0), & \text{when } r_2 > \hat{r}_2(r_1), \\ = \Pi_2(\hat{\xi}_1, 0), & \text{when } r_2 \leq \hat{r}_2(r_1). \end{cases}$$

Furthermore, by Proposition 3, $\frac{\partial \pi_{a_1}^*(q_{a_1}, q_{a_2}, q_f)}{\partial q_f} < \frac{\partial \pi_{a_2}^*(q_{a_1}, q_{a_2}, q_f)}{\partial q_f}$ and hence, $\pi_{a_1}^{*'}(\xi) > \pi_{a_2}^{*'}(\xi)$ for any $\xi \geq 0$. Therefore, $\underline{r_2} > \max\{\underline{r_1}, \tilde{r}_1\}$, and $\hat{r}_2^{\mathrm{III}}(r_1) > r_1$. Then, $\hat{r}_2^{\mathrm{I}}(r_1) \geq \underline{r_2} > \tilde{r}_1 > r_1$ and $\hat{r}_2^{\mathrm{II}}(r_1) > \underline{r_2} > \tilde{r}_1 \geq r_1$. In summary, $\hat{r}_2(r_1) > r_1$.

Now, we can characterize the equilibrium investments (ξ_1^*, ξ_2^*).

Note that, any $(\xi_1, \xi_2) > (0, 0)$ is a Nash equilibrium only if (ξ_1, ξ_2) satisfies the two equations (3) simultaneously. Then, $r_2 = [\pi_{a_2}^{*'}(r_1\hat{\xi}_1)]^{-1}$. Furthermore, when $r_1 < \tilde{r}_1$, $r_1\hat{\xi}_1 < \tilde{\xi} < r_2\hat{\xi}_2$, which implies that, even though $(\xi_1, \xi_2) > (0, 0)$ satisfies the two equations (3) simultaneously, ξ_2 is not the global maximizer of $\Pi_2(\xi_1, \xi_2)$; when $r_1 = \tilde{r}_1$, $r_2 = [\pi_{a_2}^{*'}(r_1\hat{\xi}_1)]^{-1} = \underline{r_2}$ and $\xi_2^* = 0$ for any $\xi_1 \geq 0$. Therefore, $(\xi_1, \xi_2) > (0, 0)$ can be an equilibrium if and only if $r_1 > \tilde{r}_1$ and $r_2 = [\pi_{a_2}^{*'}(r_1\hat{\xi}_1)]^{-1} = \hat{r}_2^{\mathrm{III}}(r_1)$. In this case, there exist multiple Nash equilibria (ξ_1^*, ξ_2^*) satisfying $r_1\xi_1^* + r_2\xi_2^* = r_1\hat{\xi}_1 = r_2\hat{\xi}_2$ and $(\xi_1^*, \xi_2^*) \geq (0, 0)$. Otherwise, an equilibrium can only be $(0, 0)$, $(\hat{\xi}_1, 0)$ or $(0, \hat{\xi}_2)$. $\hat{\xi}_2$ exists only if $r_2 > \underline{r_2}$.

In region 1, $\hat{\xi}_1 = 0$ since $r_1 \leq \underline{r_1}$, and then $\xi_2^* = 0$ since $r_2 \leq \hat{r}_2$. Therefore, $(0, 0)$ is the only Nash equilibrium. In the other regions, since $r_1 > \underline{r_1}$ or $r_2 > \hat{r}_2$, $(0, 0)$ cannot be an equilibrium. In region 2, when $r_2 \leq \underline{r_2}$, $\xi_2^* = 0$ and $(\hat{\xi}_1, 0)$ is the only equilibrium; when $\underline{r_1} < r_1 < \tilde{r}_1$ and $\underline{r_2} < r_2 \leq \hat{r}_2(r_1)$, $(\hat{\xi}_1, 0)$ is an equilibrium since $\max_{\xi_2>0}\{\Pi_2(\hat{\xi}_1, \xi_2)\} \leq \Pi_2(\hat{\xi}_1, 0)$, but $(0, \hat{\xi}_2)$ is not since $r_2 \leq \hat{r}_2(r_1) < \hat{r}_2$; when $r_1 \geq \tilde{r}_1$ and $\underline{r_2} < r_2 < \hat{r}_2(r_1)$, it is not difficult to verify that $\hat{\xi}_1 > \hat{\xi}_2$, and thus $(\hat{\xi}_1, 0)$ is the only equilibrium also. In region 3, $r_2 > \hat{r}_2(r_1)$ implies that $(\hat{\xi}_1, 0)$ cannot be an equilibrium, so $(0, \hat{\xi}_2)$ is the only equilibrium. Also, in region 4, $(\hat{\xi}_1, 0)$ is not an equilibrium as $r_2 > \hat{r}_2(r_1)$. However, $(0, \hat{\xi}_2)$ is not an equilibrium either because $r_2 \leq \hat{r}_2$. Therefore, in this region there does not exist any Nash equilibrium. □

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